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Evidences on Harbingers of Mensuration Methodology in Ancient Egyptian Mathematics and Geometry

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Abstract— The Ancient Egyptians created and developed effective methods for land surveying, leveling, and mensuration, and have used mathematics to deal with these methods of mensuration. Mensuration is a branch of mathematical science that is concerned with the measurement of areas and volumes of various geometric figures. Figures such as cubes, cuboids, cylinders, cones and spheres have volume and area. In the broadest sense, the mensuration is all about the process and approach of measurement that addresses the development of formulas to measure their areas and volumes. It is based on the use of algebraic equations and geometric calculations to provide factual information regarding the measurement of width, depth and volume of a given object or group of objects. Whilst the measurement results gained via the use of mensuration are estimates rather than actual physical measurements, the mathematical calculations are usually considered more accurate.

Index Terms (Keywords) — Geometry, Surveying, Leveling, Arithmetic, Mensuration, Methodology, Houses of Science.

1 Introduction

Ancient Egyptians depended on a natural method to measure dimensions such as the arm, which was used as a measure of length, approximately equal to the length of a forearm. Traditionally, it was the length from the bent elbow to the tips of the fingers. Typically, almost 18 inches or 44 cm, however there was a long cubit of about 21 inches or 52 cm. The second natural method was the width of the palm of the hand. As well as, the human fingers used as digits of measuring width. So, four digits equal the sign of one palm and seven palms equals one cubit. In present-day trigonometry, cotangent require the same units for both the horizontal run and vertical rise , however ancient sources such as Rhind Papyrus uses palms for the run and cubits for the rise, resulting in these different, yet characteristic mathematics. So, in ancient Egypt there were seven palms in a cubit, in addition to the Seked, which was seven times the cotangent. The Egyptian Seked/Seqed is the ratio of the run to the rise of a slope of a cotangent. The Rhind Papyrus - an ancient Egyptian source or document mentioned the Seked, which is the base of many problems or issues such as; 56, 57, 58, 59 ,59 b and 60.The methodology of sloping span length measuring in ancient Egypt was based on the mathematical calculation mentioned in linguistic sources, tomb scenes, temples and stelae. There are some differences of opinion which have caused confusion over how to measure the length of a cubit accurately. A.H., Gardiner mentioned that a cubit measured 20.6 inches or 523 millimeters. Further to this, H. Carter & A.H. Gardiner emphasized that a cubit measured 523 millimeters, according to information about the tomb of Ramesses IV mentioned in the Turin plan of a royal tomb. Whilst, C. Desroches-Noblecourt stated that in King Tutankhamun's tomb, four models of the famous Egyptian unit of length were found and they measured one foot, seven and one half inches. Furthermore, Budge stated that a cubit is 0.525 meters. According to S.B. Shaffer, a cubit is a measure of length, mainly the length of the forearm, from the elbow to the end of the middle finger; 18 inches or 45.72 centimeters. As stated in the Encyclopaedia Britannica/Merriam-Webster, the cubit is an ancient unit of length based on the length of the forearm from the elbow to the tip of the middle finger and usually equal to about 18 inches or 46 centimeters. It is noteworthy that, V. Naguib argued that the rod or linear measure is a full arm of 70 cm length. It is believed that the ancient Egyptians introduced the earliest well-developed counting' or numeration system by using Hieroglyphic signs containing unit fractions, cardinal and ordinal numbers, terms, issues, laws and how to solve first order linear equations belonging to arithmetic and geometry, thus there were many terms in ancient Egyptian sources.

2 The value of sciences in ancient Egypt

Ancient Egyptians well-informed in many sciences such as Geometry, Surveying [1], [2],[3],[4],[5],[6],Astronomy[7],[8],[9]and Mathematics[10],[11],[7],[3]. Mathematics during ancient times as well as the present is the science of structure, system, arrangement and relation that has evolved from the primary practices of numeration, measuring, and describing the forms of objects. It deals with rational reasoning, proofs and quantitative calculation. The development of mathematics has involved an increasing degree of idealization and abstraction of its subject matter [12].

Ancient Egyptians believed that there are some links between Mathematics and other sciences such as Astronomy, Geology, Topography and Surveying, so they tried carefully to be aware of these sciences in order to use this knowledge in an ideal way [1],[13],[4],[2],[6].Therefore, plurality narratives, literary correspondence, documents and educational resources all promote education and race in Ancient Egyptian Sciences[14],[15],[16],[17],[18],[19],[20],[21],[22],[23],[24],[25],[26],[27],[28],[29], [30],[31],[32]. As a result, Scholars and Scientists are highly significant in Ancient Egypt[33],[34],[35], [36],[37],[38],[39], [40].There are many ancient correspondences which advocate obtaining knowledge and science in ancient Egypt [41],[42],[43],[44].

3 Sources of Scientific Life in Ancient Egypt

The Houses of Science or as the ancient Egyptians called, Houses of Life "**Prw-anẖ**" were established as Centers for science education[45],[46],[47],[48],[49],[50],[51], [52].There are many titles that connected Gods with houses of life, which were used as centers of sciences, education and knowledge. Just like in Esna [53], El-Tod[54], Edfu[55]. Further evidence is found in text of the Sixth Dynasty that mentions the house of life in Al Hagarsah, located south of Sohag[56], also in Abydos[57],[58],[59], Al Barsha of EL-Minya[60],El Hiba[61],Lisht[62],Thebes[63],Heliopolis[51],[64],[65], [66],[67],[68],[69],[70],[71],[72],Memphis[73],[74],[75], [66], [76], [77],[78] and Bubastis[79]. There were other Houses of documents throughout ancient Egypt[80],[81],[37],[82],[83], [84], [51], [85], [86],[87],[88],[89],[90],[91].Late Egyptian and Greek sources praised the ancient Egyptian cultural centers as a source for knowledge and Sciences[92], which was a source of inspiration for the legislator "Solon"[93],[94],[95], as well as "Thales of Miletus" who was a mathematician and astronomer, and for this reason he learned and practiced the Geometry of ancient Egypt, then taking this knowledge to the Greeks [96],[97],[98],[99],[100],[101],[102],[103],[104],[105],[106],[107],[108],[109],[110]. As for "Pythagoras of Samos", he was a disciple of "Thales" who advise Pythagoras to complete his studies in ancient Egypt, and he then spent about twenty two years in Egypt studying astronomy and geometry[111],[109],[112]. Included in this group of ancient Greeks who studied in Egypt were "Plato"[113], and "Eudoxus"[114]. Evidence of the value of the Egyptian sciences are that the wisest of the Greeks; "Solon, Thales, Plato, Eudoxus, Pythagoras", in addition to "Lycurgus" also, came to Egypt and consorted with the priests [115],[63],[93],[116],[117],[109],[118],[119].

4 Orthographic Formulations of Phonetic Signs and Linguistic Values of Numeration and Dimensions

The invention of writing was a reflection of the life-style and the environment of the Egyptian culture as well as the cosmic aspects, which were the core of ancient Egyptian ideology, particularly the inundation cycle and the harvest cycle, the sun rise and sun set, all these cosmic phenomena were the nucleus of the Egyptian philosophy of life, death and resurrection. The ideology of life, death and rebirth engulfed the ancient Egyptian life-style and all various aspects of an individual's life on earth and after their death. This ideology was the main reason why now have what we do from the ancient Egyptian culture. Orthographic signs of Hieroglyphs were not just a form of an object, but a writing system to convey aspects of the sound and meaning of ancient Egyptian language. Each of these signs expressed a sound, some having one sound (Consonant signs), others have two sounds (Biliteral signs), or three sounds (Triliteral signs) and in some rare cases four sound were expressed. The difficulties that faced the group of ancient Egyptian pioneers who invented this writing system can only be imagined. There were so many signs of which carried a number of Phonetic values, which formed the syntax of the ancient Egyptian language. The basic principle of the Hieroglyphic orthographic system includes two major usages. The first usage is ideograms, which are signs used to convey both sound and meaning. The second usage is phonograms, which are signs used to indicate the sounds of signs. The most common Hieroglyphic signs are those which represent a single vowel or Uniliteral signs [24]. Ancient Egyptian mathematical numbers and fractions could be classified in two elements; Cardinals and Ordinals[14],[25],[120],[121],[26],[16],[123],[124],[45], [37],[125]. Cardinals are simply 1, 2, 3, etc. It is noteworthy that the higher value is written in front of the lesser value and the numeral follows the noun, which as a general rule, exhibits the singular form in the cardinal manner[14],[16],[25]. Ordinals means first, second, third, etc. Notably, the ordinals from 2-9 are signed by adding the ending (**nw**) after the number[126],[127],[128],[129],[130],[120], [131],132],[16],so you have the number + **nw** and from the ordinal number 10 upwards, the Egyptian used the word (participle) (**mḥ**) "completing" and it is written first + number and it should be noticed that all units follow their nouns. Ancient Egyptians used (**r**) to express fractions, meaning part below the number, which is used as a denominator or numerator such as; **r** $1/3$ "Third". Noteworthy, the Egyptians did not have a phonetic value for a repeated fraction unit such as; **r** $1/4$ "Quarter $1/4$"[25],[26],[120],[121],[122],[123],[16]. In Measurement, they employed the Eye of Horus and its parts in order to distinguish measures and weights. They divided the eye into 6 units and utilized them for measuring and weighing grains. Noteworthy, it starts with $1/2$, $1/4$, $1/8$, $1/16$, $1/32$, $1/64$ [25], [133],[134],[135],[136].

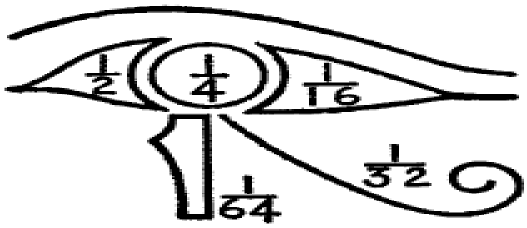


Fig.1: The Eye of the Falcon-god Horus as illustrated by mythological and symbolic beliefs of ancient Egyptians. Source: [137].

As mentioned above briefly, the measurement of dimensions is used in ancient Egypt for the objective of surveying, measuring and numbering. The royal cubit was a familiar pattern, as well as the rope used by surveyors, equal to a surveyors' chain, which was about one hundred royal cubits in length. These could be summarized in below schedule:

Length Unit	Equivalent Unit	Further Unit	Notes
Four Fingers	One Palm		
Twenty-four Fingers	Six Palms	One Cubit	
Twenty-eight Fingers	Seven Palms	One Royal Cubit	Almost 52 cm
One hundred Royal Cubits	One Khet		
Twenty thousand Cubits	One Iteru		Almost 10.5 km
One Khet × One Khet	One Aroura	One Setat	Almost 0.25 Hectare "Distances"

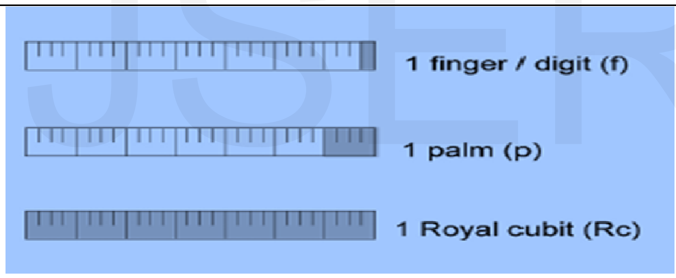


Fig.2: A diagram clarifies the ancient Egyptian units. Source:[138].

It is believed that the ancient Egyptians introduced the earliest well-developed counting' or numeration system by using Hieroglyphic signs containing unit fractions, cardinal and ordinal numbers, terms, issues, laws and how to solve first order linear equations belonging to arithmetic and geometry, thus there were many terms in ancient Egyptian sources, these could be outlined as follows:

4.1 Cubit

Pyramid texts mentioned that there are some writing forms that signify the meaning of cubit, yardstick or linear measure such as; , , **mḥ** [139],[140],[141],[120], it is also mentioned in an abbreviated form such as; , , it is almost 52 cm. [139],[142],[143],[144],[145],[146],[122]. According to Sahidic Coptic **ⲘⲀϨⲈ** and Bohairic Coptic **ⲘⲀϨⲒ**, it is signified to be masculine [147],[148]. It should be mentioned that there are some differences which have led to an element of confusion when we want to determine the length of a cubit accurately. A.H., Gardiner mentioned that a cubit measured 20.6 inches or 523 millimeters [137].Further to this, H. Carter & A.H. Gardiner emphasized that evidence that a cubit measures 523 millimeters, according to information about the tomb of Ramesses IV, which is mentioned in the Turin plan of a royal tomb [141].

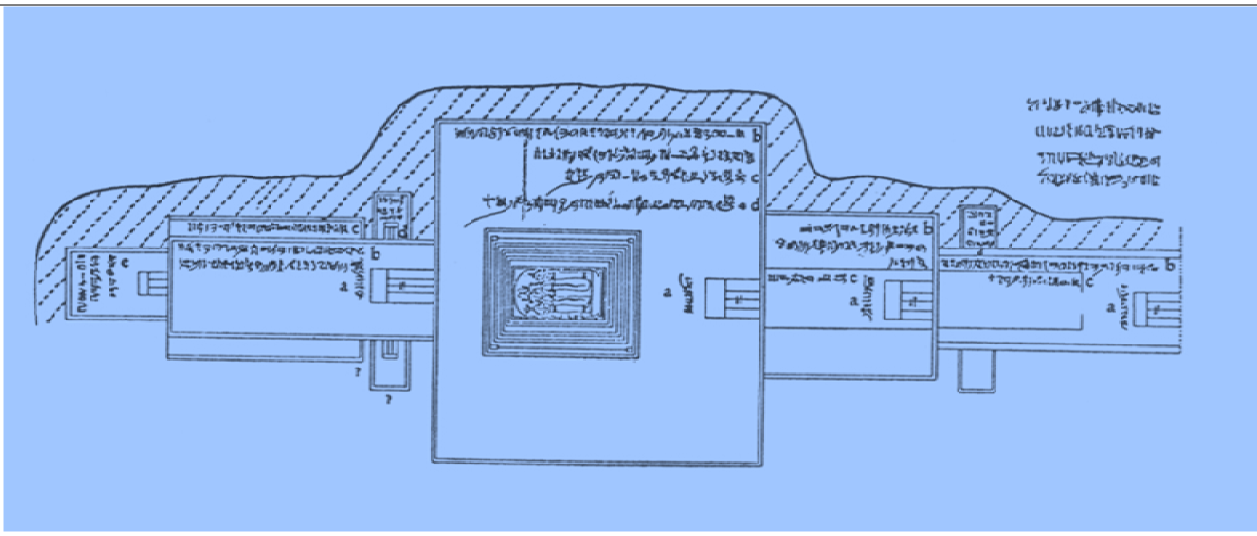


Fig.3: The dimensions of the various shafts, chambers, and niches of the tomb of King Ramses IV as depicted on the Turin Papyrus .Source: [149].

Evidence from one of the rare plans depicted on a piece of papyrus, now preserved in the Turin Museum. It illustrates a plan and lists the dimensions of the various shafts, chambers, and niches of the tomb of King Ramses IV, who reigned circa 1160 B.C. Noteworthy, the relationship of the measurements, as shown on the papyrus, and those measured in the tomb when it was first studied by Howard Carter and later by Kent Weeks of the Berkeley Theban Mapping Project/BTMP[149],[150].When Weeks compared the measurements of the papyrus and those taken by the BTMP, he used Carter's suggested value of the cubit of 1 Cubit = 0.5231 meters. There is an example of the type of results displays a mix. For instance, in Chamber C, Weeks shows the following:

Dimensions	BTMP Measurement	Turin Papyrus
Length	13.188 m	13.078 m
Width	3.14 m	3.139 m
Height	5.07 m	5.006 m

Accordingly, other chambers and passages present similar results; also, the measured lengths were shorter than the planned lengths. Precisely, the measurements of Carter's value for the cubit are close to an accepted value or correct value. The variation in dimensions may be due to the workmen who were skimping on the mission work slightly, or perhaps the plans or designs have disappeared over time [149], [150]. Whilst, C. Desroches-Noblecourt stated that in King Tutankhamun's tomb, four models of the famous Egyptian unit of length were found and they measured one foot and seven and one half inches [151]. Furthermore, Budge stated that a cubit is 0.525 meters [152]. According to S.B. Shaffer a cubit is a measure of length. Mainly, the length of the forearm, from the elbow to the end of the middle finger; 18 inches or 45.72 centimeters[153]. As stated in the Encyclopaedia Britannica/ Merriam-Webster, the cubit is an ancient unit of length based on the length of the forearm from the elbow to the tip of the middle finger and usually equal to about 18 inches or 46 centimeters [154]. Archaeological evidences emphasized that there were two types of cubits that were used, depending on the length of the model, as well as according to the method of a measurement unit that was used or what can be called the methodology of measurement. There are variants of writing forms which differ in some respect from previous forms such as; , **mḥ nsw.t** , it means royal cubit; it is a long cubit of about 21 inches or 52 cm [155],[156],[157]. Another writing form **mḥ šrr / mḥ nḏs** , it means small or low cubit, it was $^{6}/_{7}$ of normal or standard cubit and it is almost 18 inches or 44 cm[157],[156].

4.2 Palm

The Old Kingdom sources mentioned that there are some writing forms that signify the meaning of the length dimension of four fingers or the linear measure of one palm or palm-breadth such as; , **šsp** [35]. It is reported in middle and new kingdom sources as the as following forms; , , , [158],[159], [160], [10],[17],[124]. It also mentioned in an abbreviated form such as; , [124], [137/Sign-list, under D48].

4.3 Digit

The Old Kingdom sources mentioned that there are some writing forms supporting the meaning of digit, i.e. finger-breadth or the length dimension of one finger including the thumb or each of the four slender jointed parts attached to either hand [161], [162],[163],[16],[146],[132],[164]. Noteworthy, is the linear measure of four fingers equivalent to one palm as aforementioned. The Pyramid texts reported the writing forms as follows; **ḏb'**[81/Spells 734;891;1113; 1278;1330;1837]. The variant writing form in sources of the Old kingdom are in the singular; , and in the plural; . The Middle kingdom sources reported that the singular form as follows; , and in the plural; . The sources of the New kingdom reported the singular form as follows; , and in the plural; ; [165],[166]. To a smaller extent, a frequently mentioned linear measure was that the "pole", may be equal in value to $1\frac{1}{4}$ or $1\frac{1}{3}$ cubit [167],[168],[169]. Its writing forms were as follows; , , , , **nbiw** [170],[137],[171],[172],[131/ P. Ebers, 49,2-3;54, 22; 98a,19].

4.4 One Hundred Royal Cubits

The Middle kingdom sources reported that there were writing forms which indicated the meaning of One Hundred Royal Cubits, was similar to a thin straight bar, especially of wood which was used as a measurement rod or a linear measure[45], [10/P.Rhind,49],[16/Urk.IV,133],[173]. Its writing forms were as follows; , [165/Wb,III,341,12].The Pyramid texts and other sources from the Old and Middle Kingdoms reported that the afore- mentioned term, associated with other writing forms were as follows; , , , **nwḥ**[81/Spell 138],[174],[175],[176], [177],[178],[179]. Figuratively, it is used in the expression of **ẖt n nwḥ**, which means a linear measure of 100 royal cubits [16/Urk, IV, 133]. In Ptolemaic sources, there were variant writing forms such as; , [170/Wb,II,223,12]. Furthermore, it was attached to another writing form in the expression; **ḥat nwḥ**, it means "the beginning of a linear measure of 100 royal cubits"[155],[180], [17]. A much larger linear measure was the river-measure, the Greek "Schoenus", it estimated the range almost 20, 000 Cubits=10.5 km [181],[182],[183],[180/Amarna V, Nos. 8, 18-19].

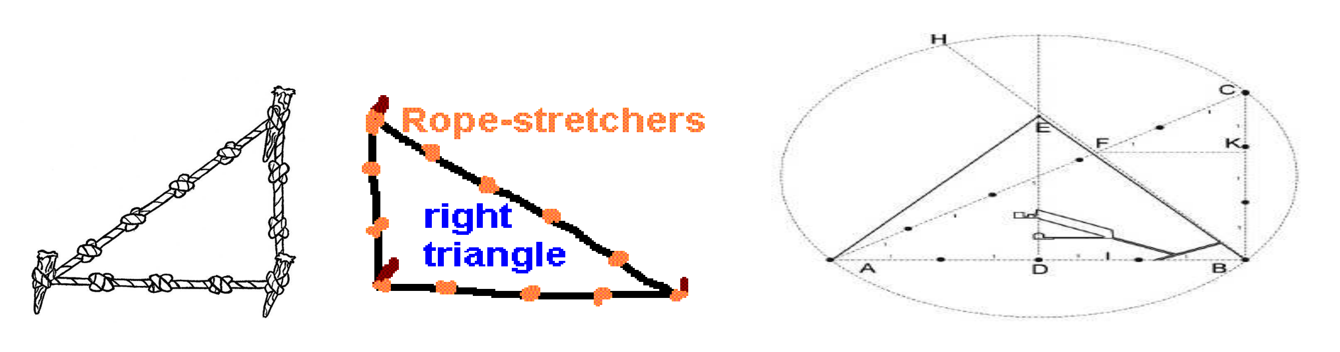
4.5 Square Khet or Equivalent of One Aroura and One Setat

The ancient Egyptian linguistic sources reported that there are some writing forms which indicate the meaning of 100 Cubits squared, i.e. 2735 Square meters or almost $\frac{2}{3}$ Acre. Moreover, it is a unit of land area equal to 4,840 square yards (0.405 Hectare). Its writing forms were as follows; , , , , **sṯat**, the Greek "Aroura"[137],[184],[185], [186],[133], [25], [16/Urk,IV,6,8.15],[120],[124],[10],[187]. There were fractions of the **stat** such as the ideogram that is equal in value to 10 arouras, which was in the form **ẖa**, literally "thousand", the full form was **ẖa-ta**[188],[120/ No. I, 313, 325],[124/

P. Kahun, 21,3;21,19],[10/P. Rhind, Nos. 48,53;54,55]. The ideogram **rmn** was equal in value to $\frac{1}{2}$ **stat**, in addition to the ideogram **sa** that was equal in value to $\frac{1}{8}$ **stat.** Furthermore, there were minimal parts of the aroura that were expressed in the ideogram **mḥ**=cubit, i.e. a sector of a zone that equals 100 cubits in length and 1 cubit in depth= $\frac{1}{100}$ **stat** or aroura [133], [25],[173],[189],[190],[191].

5 Evidences on Inclusiveness in a Metaphorical Sense within Mensuration Methodology in Ancient Egypt

Ancient Egyptian sources such as Pyramid texts [81], Pap. Ch. Beatty [192], Limestone plate in Cairo museum (J.d'E.No.671000) [80], Pap. Rhind[193],[187], [194], and other sources included many aspects and issues of surveying related to mathematics[195], and measurement, orientation, leveling and legal issues of surveying, like; rectangle, triangle, triangle ellipse, half circle, and the solutions for those issues[196],[197],[198],[199]. The Rhind Papyrus, dating back to almost 1650 B.C, was an important source of sciences of arithmetic and geometry, and it lets us to know how the multiplication and the division were achieved in ancient times. It is also includes clues of mathematical and Geometrical information, including how to resolve the issue of first degree linear equations beside issues of arithmetic and geometric . Furthermore, the Berlin Papyrus, which dates back to about 1300 B.C, and refers to how to resolve the issue of second-degree of algebraic quadratic equations by ancient Egyptians. Notably, most of the existing evidence comes from the paintings on tomb walls or fragments of papyrus, all of these evidenced that ancient Egyptian surveyors exhorted the best and used the best methodology for surveying [200],[201], [202],[203],[204],[205]. In Egypt, Pythagoras studied with the people known as the rope-stretchers. The rope stretchers were the surveyors of land and buildings. These people were the engineers who built the pyramids. It is noteworthy that in order to determine the intersection point of two lines by extending them indefinitely - a method that can be compared to the sighting of points and the measurement of geometric forms using a dioptra, an ancient surveying instrument that Euclid mentioned in his works on astronomy[206],[207],[208]. Euclidean geometry was a measurement method through logical proofs and based on the intellectual meditation. It is clear that the theory of Euclid was a proof of geometric hypothesis resulted from ancient Egyptian geometry, which indicates to physical reality. The Egyptian geometers used a rope divided into equal parts, later known as a Pythagorean triangle to measure a right angle. The rope method enabled them to compare angles, even those that were called the non adjacent angles. The construction of triangles was one of the most significant tasks in land surveying [208],[206],[207].The rope-stretchers had other types of ropes, there was one shape of a rope which was tied in a circle with 12 equally diverged knots. It becomes clear that if the rope was pegged to the ground in the dimensions of 3-4-5, a right triangle would elicit. This enabled the ancient Egyptian builders to design the foundations for their structures precisely.

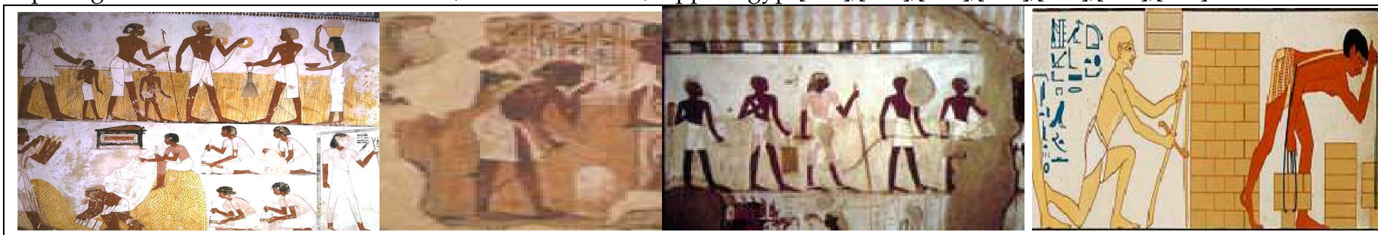


(A) (B) (C)

Fig. 4: (A) A simple diagram of a Pythagorean triangle inspired by the idea of the ancient Egyptian triangle. Source: [208]. (B) Design illustrates the diagram (A). The Diagram (C) illustrates the developing methodology of a Mathematical theory of the triangle. Source: [138].

Ancient Mathematicians have used multiplication and divisions which were carried out by facilitating the numbers so that only two or ten had to be multiplied, being the length measures used for the objectives of surveying [133]. Ancient surveyors also used this for some aspects, including determination and definition of topographical or geographical borders, as well as in building construction [204],[209],[210],[211],[212],[213], [214],[215]. The Nile flood was an influence on the features of Egyptian life, quite often resulting in a change of the form and terrain of the land. So, it then required a surveyor in order to re-measure or re-survey the land[216],[217],[218],[219],[220].In ancient Egypt, the checking of boundaries was as an allocating method of the environment, which was the major mission of the geometer, literally; "the measurer of the earth" [208], [221],[207],[222].

It is noteworthy that the measurement of the hypotenuse was an important aspect of ancient Egyptian mathematics and geometry which was a practical method in the land surveying, leveling"[208],[221], [222]. The surveyor had an important role, as shown by the evidences of the work of the surveyors and surveying in the approach of representation scenes on tomb walls. Scenes of surveying in the fields have been found on tomb walls[223], such as the scene of a surveyor checking a boundary line as in the tombs of the Nobles "Menna, Nebamun and Djeserkeresonb", and the scenes of measuring the level of the structure slope angle as in the tomb of "Rekhmire" , found in Thebes, Upper Egypt[201],[205],[208],[221],[207],[222],[224].



(A) (B) (C) (D)

Fig. 5: (A) Surveying of the land with the aim of calculating the taxes from the tomb of Menna. Source: [225]. (B) A surveyor in the process of checking the borders from the tomb of Nebamun. Source: [201]. (C) Surveyors checking the boundaries and in the process of surveying of the land from the tomb of Djeserkeresonb. Source:[208]. (D) A measuring tool used to gauge the level of the structure slope angle from the tomb of Rekhmire, Thebes, Upper Egypt. Source: [224].

The position of the surveyors displays that they were one of the higher caste and well educated within Egyptian society [226],[227],[228]. The science of surveying was a great function of the state and therefore revolved around careful, accurate accounts which allowed registration, documentation and calculation of the land of the King who theoretically owned all the land, and delegated its use to others[204],[229]. Since the Third Dynasty upwards, the land was given to deserving officials [205], which eventually led to the land being owned mostly by temples or individuals, who were then required to pay tax on the land. There are records that date back to around 3000 B.C of the registration of land [229], [228], [230], [231]. The methodology used for this system of land management was to require an assessment of land ownership in order to calculate the tax. Due to the constant changes in the land, it was the job of the surveyor to measure each parcel of land annually so that the tax could be calculated [200], [203]. To produce level surfaces meant careful specifications of some practiced methods or tools used by ancient Egyptians surveyors such as Nilometers which are defined as graduated pillars or other vertical surfaces, serving to indicate the height to which the Nile rises during its annual floods, also used for calculating the modification of the land. For leveling, they used a simple frame shaped level with a plumb bob hung, which marked the centre point and then by turning it into 180 degrees could even determine the level, in this case the pillars were unmatched or incomparable the extent of length [232],[233], [234], [235], [236],[237]. A gnomon also used by surveyors and formed by a vertical pillar that they could use to measure the shadows or to mark out the orbit or path of the sun [238],[239],[240],[241]. A measuring bar or rope that was symmetrical to the chain used to record the size range of zones. The chain or rope, often depicted coiled, was almost 100 cubits long, a knot was used to measure each cubit [139],[142], [143], [144], [145], [146], [202], [155],[156],[157]. The method or methodology for the measuring of a sloping span length in ancient Egypt becomes clear via religious and funeral structures. A very impressive structure is the great pyramid of King Khufu in the Giza plateau [207], [242],[243],[244],[245],[246], [2],[247],[248],[9]. The most important aspect of pyramid construction was the precise mathematical calculation and the determination of the sloping span length of a structure, which was carefully kept close to the line of the horizon. Noteworthy, there is a belief that the metrological standards in ancient eras matched the same principles that are used in the present; therefore, the result of a unit would have measured accurately the same absolute value whatever the context. It would become clear that in the field of monumental architecture units were consistent only within the framework of a particular mission. So, a certain set of standards for each pyramid be created and ritually dedicated specifically for that pyramid [249], [250]. The average width of the base of the Great Pyramid is about 370 Horizon cubits "pyk belady", the variance being nearly 1 part in $6{,}000$ or 0.016%, here the approximation figure will be supposed to be the planned or the desired width. The pyramid ratios apparently were matched with the proportions or values of a Pythagorean [3-4-5] triangle, giving a theoretical height of $246\,{}^{2}/_{3}$ Cubits or 370 Feet. The Seked of a pyramid is calculated by finding $\mathbf{x}$ in terms of $\mathbf{y}$ then multiplying the coefficient of the determination of $\mathbf{y}$ by 7. In another meaning it is 7 times the cotangent of the pyramid's dihedral angle. Subsequently, for the Horizon Pyramid, the slope considered as $\mathbf{y}$ in terms of $\mathbf{x}$ is $\mathbf{y} = 14^{x/11}$. As a result of this, the slope expressed as $\mathbf{x}$ in terms of $\mathbf{y}$ is $\mathbf{x} = 11^{y/14}$, making the cotangent of the dihedral angle ${}^{11}/_{14}$.Multiplying this by 7 gives ${}^{11}/_{2}$ or $5\,{}^{1}/_{2}$. In order to determine the slope of an angle, it should be considered an angle formed by a horizontal line and a slanted line as in the illustration below. Such an angle can be specified by just giving its slope. In the illustration, the slope of angle A is the ratio h/b.In geometry, this quantity is also referred to as the tangent of the angle A and denoted by Tan (A).The angle A can also be specified by Cot (A) =b/h, which is called the cotangent of the angle A and is just the inverse of the slope [251].

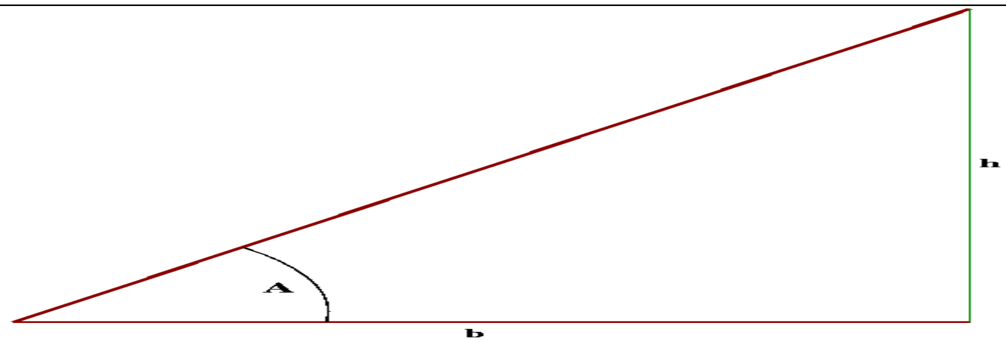


Fig.8: The diagram illustrates angle A, which can also be specified by Cot (A) =b/h, which is called the cotangent of the angle A and is just the inverse of the slope). Source: [252].

6 Conclusion

Dimension is the mensuration of an item that can be seen and touched and have a specified size or extent such as length, width, and height. Dimensional measures include a one-dimensional concept /length, two-dimensional concepts/length and width or three-dimensional geometric figures/ height, width and depth. Mathematically, it is derived from a concept that a line is one-dimensional, a level is two-dimensional, and size is three-dimensional. In physics and also in mathematics there is a point of view which can be called a higher-dimensional extent such as four-dimensional space-time, which should be four digits with a view to typify an element; three to rectify a point in the range of space and one to rectify the scope of time. Furthermore, the infinite-dimensional scopes have played an even more significant role in quantum field theory. The distance formula is defined to give the range between pairs of points in terms of their coordinates in Algebraic system. Comparably, in ancient Egypt and here, there are many mathematical issues that examine aspects of mathematics, measurement, orientation, leveling and surveying. Mathematics in ancient Egypt consists of an ascending series of digits. Multiplication and Division were implemented by simplifying the digits. In ancient Egypt, Pythagoras studied with the so-called, rope-stretchers, and they were the surveyors. Those people were the engineers who built the pyramids. It is believed that the ancient Egyptians elaborated the earliest fully-developed base numeration system about 2700 B.C. they used a stroke for units, a heel-bone sign for tens, a coil of rope for hundreds and a lotus plant for thousands, beside the other Hieroglyphic signs for higher digits of ten to about a million up to the infinite symbol or sign. Euclid's aspects are a typical form of ancient intellect, in view of these parallels between practical experience and theoretical abstraction; it should be assumed that, for Euclid, the evident nature of the geometric postulations resulted from geometrein or from the constructive act of measuring land. The Egyptian geometers used both a rope divided into equal segments and a Pythagorean triangle to measure a right angle. A framework of this rope system enabled them to compare angles, even those that were non-adjacent. Along with the formation of straight lines and circles, the construction of triangles was one of the most important tasks in land surveying, and it is described in the Fifth postulate of Euclidean geometry. Furthermore, there is ancient Egyptians system that can be compared to the vision of points and the measurement of geometric shapes using a dioptra system, which is an ancient surveying tool that is mentioned in Euclidean astronomy. Rationally, the basic precedents for the mathematical efforts of the Greeks and how dealing with fractional segments or measured areas, extents and levels, or how the use of ratios, is believed to be derived from the knowledge and sciences of the ancient Egyptians. In ancient and present times, mathematics has played an even more significant role in architecture. The relevance between architecture and mathematics interacted in ancient Egypt and also in other cultures. It is a system that has been devised to measure time, distance, area, volume, weight, and units that measure these quantities. Recently, a system of measurement which has been recognized to be valid all over the world is the standard system for use in science and trade known as the International System of Units/SI. It is believed that the ancient Egyptians geometers were typically using units of measure similar in value and carefully related to each other. As mentioned above, there are some differences which cause some confusion when we want to measure the length of a cubit or any of the various ancient units of measurement, dimensions and sloping span length accurately. There is variation in opinions of scientist such as what has been mentioned by "Gardiner, Carter & Gardiner, Noblecourt, Budge, Shaffer, Naguib and Encyclopaedia Britannica/ Merriam-Webster". It should certainly be considered that there were also variations of what stated in the sources of ancient Egyptians. Thus, some confusion may have originated in Egypt close to 5,000 years ago. The cubit can refer to various units used in the ancient world, the actual length of which varied from time to time and place to place, but which was generally equivalent to the length of the human arm from elbow to fingertip-roughly about a foot and a half. The word's source is a Latin word meaning "elbow", starting with the Wycliffe Bible in 1382, the cubit has been used as the

English translation for the measurement known in Biblical Hebrew as the ammah and in Koine, which means "common" or "shared" in Greek. Greek was the language spoken in the eastern Mediterranean countries from the 4th century B.C. until the time of the Byzantine emperor Justinian almost mid-6th century A.D., this dialect or language was from a region that became the common or standard language centre of a larger area, principally the Greek language which was commonly spoken and written in the eastern Mediterranean countries during the Hellenistic and Roman periods, where they knew the cubit as the " Péchus ". Mostly, in sciences, religions, habits, traditions and in applied methodologies, mainly in linguistics the word is applied to a language developed from contact between dialects of the same language over a large region. Basically, any word adopts those grammatical and lexical elements from the dialects of the region that are more easily recognized by the speakers of that area. They then dispense with those elements that are not. Accordingly, there are differences and disparities in the methods of measurement. It is therefore, an important reason to think carefully about orientation methodology of archaeological locations and sites of excavation. Thence, there should be differences shown from one place to another during the epochs and ages. This explains the disparity between any ancient idea and the extent of its transition and development of cultural inheritance during the sequence of historical eras. This is the purpose of the discussion above, which depended on ancient Egyptian sources in order to illustrate the measurement methodologies of dimensions used in Ancient Egypt for the objectives of numbering, measuring of dimensions and how to measure the sloping span length in ancient Egyptian mathematics.

7 Results

7.1 Ancient Egyptians excelled in many sciences such as Geometry, Surveying, Astronomy and Mathematics. Ancient Egyptians believed that there are some links between Mathematics and other sciences such as Astronomy, Geology, Topography and Surveying, so they tried carefully to be aware of these sciences in order to use this knowledge in the best way possible.

7.2 Mathematics during ancient and present times is the science of structure, system, arrangement and relation that has evolved from primary practices of numeration, measuring, and describing the forms of objects and issues.

7.3 Ancient Egyptians had created and developed effective methods for land surveying, leveling, and mensuration, and hired the mathematicians to deal with the methods of mensuration. Mensuration is a branch of mathematics that is concerned with the measurement of areas and volumes of various geometric figures. In the broadest sense, mensuration is all about the process and approach of measurement that addresses the development of formulas to measure their areas and volumes.

7.4 Mensuration is based on the use of algebraic equations and geometric calculations to provide factual measurement information regarding the width, depth and volume of a given object or group of objects. Whilst the measurement results gained via the use of mensuration are estimates rather than actual physical measurements, the mathematical calculations are usually considered more accurate.

7.5 Most of the existing evidence comes from the paintings on the tomb walls or fragments of papyrus, all of these evidenced that ancient Egyptian surveyors created and used the best methodology for surveying. In Egypt, Pythagoras studied with the people known as the "rope-stretchers". The rope stretchers were the surveyors of land and buildings. These people were the engineers who built the pyramids. Noteworthy, in relation to determining the intersection point of two lines by extending them indefinitely - a method that can be compared to the sighting of points and the measurement of geometric forms using a dioptra, an ancient surveying instrument that Euclid mentioned in his works on astronomy.

7.6 The methodology of sloping span length measuring in ancient Egypt was based on the calculation of mathematics that is mentioned in linguistic sources, scenes of tombs, temples and stelae.

7.7 Multiplication and divisions were carried out by ancient mathematicians to facilitate the numbers so that only two or ten had to be multiplied. These were the length measures used for the desired outcomes of surveying.

7.8 There are some differences which have caused confusion when we want to measure the length of cubit accurately. The variation in opinions of scientists such as those mentioned by "Gardiner, Carter & Gardiner, Noblecourt, Budge, Shaffer, Naguib and Encyclopaedia Britannica/ Merriam-Webster". It should certainly be considered that there were also variations of what was stated in the sources of the ancient Egyptians. Thus, the variations may have originated in Egypt close to 5,000 years ago.

7.9 In ancient Egypt, there were seven palms in a cubit; in addition to the Seked was seven times the cotangent. The Egyptian Seked/Seqed is the ratio of the run to the rise of a slope of the cotangent. The Rhind Papyrus - an ancient Egyptian source or document mentioned the Seked, which is the base of many problems or issues such as; 56, 57, 58, 59 ,59 b and 60.

7.10 In present-day trigonometry, the cotangent requires the same units for both the horizontal run and vertical rise; however, the papyrus uses palms for the run and cubits for the rise, resulting in different yet characteristic mathematical numbers.

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